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**DIRECT AND INVERSE PROBLEMS OF HEAT TRANSFER IN SOIL**

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**Abstract.** The procedures for determining soil thermal diffusivity coefficient based on the solution of inverse problems of heat transfer equation taking into account the boundary conditions on the surface which are described by two harmonics have been developed. These procedures enable estimating the thermal diffusivity in soil under natural conditions and that should increase the adequacy and expand the use of mathematical models of soil thermal regime.

**Keywords:** soil, modeling, heat transfer, boundary conditions.

**INTRODUCTION**

To obtain comprehensive knowledge of soil thermal properties one should have the data which enables finding the values of the thermal characteristics for the specific composition and condition of the soil. The main thermal characteristics of soil are the coefficients of thermal conductivity, thermal diffusivity and thermal capacity. The knowledge of these soil characteristics may advance the solution of the topical problem of our time as forecasting soil thermal regime. In solving many problems related to soil thermal processes one should deal with soil thermal diffusivity coefficient ( $\kappa$ ). The determination of soil thermal diffusivity coefficient was discussed in many theoretical and experimental studies [1-4].

To determine soil thermal characteristics, two main groups of methods are used: computational and experimental methods. Some researchers consider the computational methods for determining thermal diffusivity and thermal conductivity coefficients to be the simplest and most convenient. Most often this is the temperature wave analysis method [2]. In most cases this method deals with the solution of heat transfer equations obtained without an initial condition and provided that  $T(\infty, t) = T_0$ . However, when performing practical calculations, it is impossible to

set the soil temperature values at infinity as the initial values since they are unknown. Therefore, usually in such cases instead of  $T(\infty, t)$  the temperature at a certain depth  $x = L$  should be set, starting with that, when  $x > L$ , the value of  $T(x, t) = \text{const}$ . Therefore, the calculation of the coefficient  $\kappa$  by the equations obtained at the solution of a heat transfer model provided that  $T(L, t) = T_0$  at the lower boundary in the soil is of interest.

The goal of this work is to develop the procedure for determining soil thermal diffusivity coefficient ( $\kappa$ ) based on the solution of inverse problems of heat transfer equation with follow-on comparison of the existing methods. The calculation results were tested on some soil types of the Konya Province.

**MATERIALS AND METHODS**

*Setting the objective and the choice of heat transfer model in soil*

To analyze the location of the temperature field in a soil profile, one may not apply the equation system of conductive, radiation and mass-exchange conductivity and use only the thermal conductivity equation with regard for the known heat transfer coefficients [4, 5]:

$$c_v(x, t) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ \lambda(x, t) \frac{\partial T}{\partial x} \right]$$

It is possible to significantly simplify this equation if we take as constant the thermal capacity coefficient and the coefficients of thermal conductivity and thermal diffusivity into the soil depth. In this case, one-dimensional heat distribution in soil is described by the classical thermal conduc-

tivity equation which (in the absence of phase transitions of soil moisture and heat transfer with the moisture and with the assumption that the thermal gradients are associated with vertical heat transfer only, and in the absence of internal sources) is as follows [1, 3, 6, 8-11]:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}, \quad \left( \kappa = \frac{\lambda}{\rho_b C_m} = \frac{\lambda}{C_v} \right) \quad (1)$$

and its analytical solutions obtained without initial condition and at periodic

boundary conditions on the surface are considered, i.e.:

$$T(0, t) = \varphi(t) = T_0 + \sum_{j=1}^m T_j \cdot \cos(j\omega t + \varepsilon_j) \quad (2)$$

and provided that the soil temperature at the

lower boundary (at infinity) is constant, i.e.:

$$\lim_{x \rightarrow \infty} T(x, t) = T_0 \quad (3)$$

Here,  $T(x, t)$  is the soil temperature at the point  $x$  at time point  $t$ ;  $\lambda$  is the thermal conductivity coefficient;  $c_v$  is the volumetric thermal capacity;  $\rho_b$  is the soil density;  $\kappa$  is the thermal diffusivity coefficient;  $T_0$  is the average daily (or yearly) the temperature of soil active surface;  $T_j$  is the oscillation amplitude of soil active surface

temperature;  $\omega = 2\pi/\tau_0$  is the circular daily (or yearly) frequency;  $\tau_0$  is the period (length) of wave expressed in days or years;  $\varepsilon$  is the phase shift depending on zero-time reference.

The solution of the problem (1)-(3) in dimensionless variables is as follows [1, 12-14]:

$$T(y, \tau) = T_0 + \sum_{j=1}^m \Phi_j(y, b_j) \cdot \cos[j\bar{\omega}\tau + \varepsilon_j - \psi_j(y, b_j)] \quad (4)$$

where  $y = x/L$ ,  $\tau = \kappa t/L^2$ ,  $b_j = \sqrt{j\bar{\omega}/2}$ ,  $\bar{\omega} = \omega L^2/\kappa$  and

$$\Phi_j(y, b_j) = T_j \cdot e^{-b_j y}, \quad \psi_j(y, b_j) = b_j y \quad (5)$$

However, when performing practical calculations, it is impossible [7] to set the soil temperature values at infinity as the initial values since they are unknown.

Therefore, in such cases instead of (3) the condition at the lower boundary is set in the following form, which describes the heat transfer process more realistically:

$$T(L, t) = T_0 \quad (6)$$

$$\Phi_j(y, b_j) = T_j \cdot \sqrt{\frac{\text{ch}(d_j) - \cos(d_j)}{\text{ch}(2b_j) - \cos(2b_j)}} \quad \psi_j(y, b_j) = \arctan \left[ \frac{Y_{2j}(y, b_j)}{Y_{1j}(y, b_j)} \right] \quad (7)$$

where

$$\begin{aligned} Y_{1j}(y, b_j) &= \text{ch}(q_j) \cos(b_j y) - \text{ch}(b_j y) \cos(q_j), \quad d_j = 2b_j(1-y) \\ Y_{2j}(y, b_j) &= \text{sh}(q_j) \sin(b_j y) - \text{sh}(b_j y) \sin(q_j), \quad q_j = b_j(2-y) \end{aligned} \quad (8)$$

$$\text{ch}(z) = (e^z + e^{-z})/2, \quad \text{sh}(z) = (e^z - e^{-z})/2 -$$

and are hyperbolic cosine and sine respectively.

The study of soil average temperature is also important since, like other soil characteristics, the temperature values vary with the depth to a lesser extent than the temperature values at a certain depth.

We define the average temperature in the layer  $0 \leq y \leq 1$ . To do this, we integrate the solution (4) from zero to one by the variable  $y$  and obtain the *mean integral solution* of the equation (1) as follows:

$$\bar{T}(\tau) = \int_0^1 T(y, \tau) dy = T_0 + \sum_{j=1}^m M_j(b_j) \cdot \cos[j\bar{\omega}\tau + \varepsilon_j - \hat{\psi}_j(b_j)] \quad (9)$$

where  $M_j(b_j)$ ,  $\varphi_j(b_j)$  and  $\hat{\psi}_j(b_j)$  under the boundary conditions (2) and (3) are determined by:

$$M_j(b_j) = T_j \cdot \sqrt{\frac{\text{ch}(b_j) - \cos(b_j)}{b_j^2 e^{b_j}}} \quad \hat{\psi}_j(b_j) = \arctan \left[ \frac{1 - e^{-b_j} [\sin(b_j) + \cos(b_j)]}{1 + e^{-b_j} [\sin(b_j) - \cos(b_j)]} \right] \quad (10)$$

under the boundary conditions (2) and (6) are determined by:

$$M_j(b_j) = \frac{T_j \cdot \sqrt{\text{sh}^2(b_j) + \sin^2(b_j)}}{\sqrt{2} b_j [\text{ch}(b_j) + \cos(b_j)]} \quad \hat{\psi}_j(b_j) = \arctan \left[ \frac{\text{sh}(b_j) - \sin(b_j)}{\text{sh}(b_j) + \sin(b_j)} \right] \quad (11)$$

The special cases of the solutions (4) and (9) with (7) and (10)-(11) are presented in the studies [3, 8, 12].

*The solution of the inverse problem of heat transfer in soil*

If the temperature of the soil surface within a day (a year) may be expressed by one harmonic, then the thermal diffusivity coefficient  $K$  may be found from the value of the daily temperature amplitude reduction with the depth or by the temperature

wave phase lag at different depths [1, 4, 11, 15]. This determination results in significant errors because the soil temperature does not always vary strictly sinusoidally. The introduction of the second harmonic (2) advances the soil active surface temperature variation to the actual state. By using the solution (4) and (7) for  $m=2$ , the equation to determine the thermal diffusivity coefficient  $K$  for arbitrary period  $\tau_0$  and dimensionless depth  $y$  may de-

rived. For this to be possible, the temperature distribution in the soil layer  $[0, L]$  for *eight time points* in the computational

time interval  $\tau_0$  should be known. Next, by using the solution (4) for  $m = 2$ :

$$T(y, \tau) = T_0 + \Phi_1(y, b_1) \cdot \cos(\bar{\omega}\tau + \alpha_1) + \Phi_2(y, b_2) \cdot \cos(2\bar{\omega}\tau + \alpha_2) \quad (12)$$

first for an arbitrary dimensionless depth  $y$  and time  $t_i = l \cdot \tau_0 / 8$  the following eight equations should be written:

$$T(y, t_i) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4}i + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2}i + \alpha_2\right), \quad (i = \overline{1, 8}) \quad (13)$$

since the following is the case

$$j\bar{\omega}\tau_i = j \cdot \frac{\omega L^2}{\kappa} \cdot \frac{\kappa}{L^2} t_i = j \cdot \omega t_i = j \cdot \frac{2\pi}{\tau_0} \cdot i \cdot \frac{\tau_0}{8} = j \cdot \frac{2\pi}{8} \cdot i \quad \bar{\omega}\tau_i = \frac{2\pi}{8} \cdot i = \frac{\pi}{4} \cdot i, \quad 2\bar{\omega}\tau_i = \frac{\pi}{2} \cdot i$$

On some rearrangement of the equations (13) we obtain [10], (see. Appendix below):

$$\sum_{i=1}^4 [T(y, t_i) - T(y, t_{i+4})]^2 = 8\Phi_1^2(y, b_1) \quad (14)$$

Taking into account both values (5) and (7)-(8) for the function  $\Phi_1(y, b_1)$  in the equation (14) we have the following expressions which correspond to the boundary conditions (3) and (6):

$$\frac{\sum_{i=1}^4 [T(y, t_i) - T(y, t_{i+4})]^2}{8T_1^2} = e^{-2b_1 y} \quad (15)$$

$$\frac{\sum_{i=1}^4 [T(y, t_i) - T(y, t_{i+4})]^2}{8T_1^2} = \frac{\text{ch}[2b_1(1-y)] - \cos[2b_1(1-y)]}{\text{ch}(2b_1) - \cos(2b_1)} \quad (16)$$

Similarly, based on the solution (9) for  $m=2$  we may derive a mean integral equation to determine the thermal diffusivity coefficient  $k$  for arbitrary period  $\tau_0$ .

First we rearrange the solution (9) for two harmonics, i.e.  $m=2$ , and then similarly to the derivation of the equations (12) and (13) we obtain the following equation:

$$\sum_{i=1}^4 [\bar{T}(t_i) - \bar{T}(t_{i+4})]^2 = 8M_1^2(b_1) \quad (17)$$

The functions in the right-hand side of the equations (14) and (17), i.e.  $\Phi_1(y, b_1)$  and  $M_1(b_1)$  depending on the boundary conditions are determined respectively from (5) and (7) and (10)-(11).

There are several procedures to determine the parameter  $k$  from the output curve of dimensionless soil temperature  $T(y, \tau)$  or  $T(\tau)$  [1, 2, 4, 9, 12, 18] expressed by the equations (4) or (9) of a soil profile.

In more detail these procedures are described by Mikayilov and Shein (2010) for the case when the soil surface temperature is expressed by one harmonic.

In this study we propose the determination of soil thermal diffusivity coefficient  $k$  based on the solution of inverse problems of heat transfer equation for the case when the soil surface temperature within a day (a year) may be expressed by two harmonics.

To determine the thermal diffusivity coefficient  $k$  (using the equations (15) and (16)), the following should be known:  $T_1$  - the oscillation amplitude of soil active sur-

face temperature;  $\tau_0$  - the period (length) of a daily (yearly) wave expressed in days or years;  $T(y_*t_i^*)$ ,  $(i=\overline{1,8})$  - the temperature values of the soil layer  $[0, L]$  at arbitrary depth  $y_*=y=x_*/L$  for *eight time points*:  $t_i^* = L \cdot \tau_0^*/4$  ( $i=\overline{1,8}$ ). For example, if  $\tau_0^* = 24$  hours, then  $t^* = 3, 6, 9, \dots, 24$  hours. Having this data, first we calculate the differ-

ences:  $[T(y_*, t_i^*) - T(y_*, t_{i+4}^*)]$  for all  $i = \overline{1,8}$ . Then from the equation (15) we obtain the value of thermal diffusivity coefficient at the depth  $x_* = x$  by the equations:

$$\kappa^* = \frac{\pi}{\tau_0} \cdot \frac{(2x_*)^2}{\ln^2 \frac{\sum_{i=1}^4 [T(x_*, t_i^*) - T(x_*, t_{i+4}^*)]^2}{8T_1^2}} \quad (18)$$

since the following is the case:

$$y_* = \frac{x_*}{L}, \quad b_1 = \sqrt{\frac{\omega}{2}} = \sqrt{\frac{\omega L^2}{2\kappa}}, \quad \omega = \frac{2\pi}{\tau_0}, \quad b_1 = L \sqrt{\frac{\pi}{\tau_0} \cdot \frac{1}{\kappa}} \quad \text{and} \quad 2b_1 y_* = 2x_* \sqrt{\frac{\pi}{\tau_0} \cdot \frac{1}{\kappa}}$$

The determination of  $k$  using the equation (16) is performed by computer-mediated fitting the values of  $b_1^*$  parameter provided that the values of the left-hand side coincide with the right-hand side calculated from the given data, i.e.

From the relation  $b_1^* = \sqrt{\omega L^2 / 2\kappa}$  we find the value of the thermal diffusivity coefficient  $k$  at the depth  $x_* = x$ , and it equals to

$$\kappa^* = \frac{\pi}{\tau_0} \cdot \left( \frac{L}{b_1^*} \right)^2 \quad (19)$$

By using the mean integral solution (9) it is also possible to find the thermal diffusivity coefficient  $k$ ; its experimental basis is the data on the temperature in the soil layer  $[0, L]$ , that is  $T(t_i)$ , and  $T_1$  as well.

In this case, the fit of the value of  $b_1^*$  parameter is performed using the equations which conform to the boundary conditions (3) and (6) respectively:

$$\frac{\sum_{i=1}^4 [\bar{T}(t_i) - \bar{T}(t_{i+4})]^2}{8T_1^2} = \frac{\text{ch}(b_1) - \cos(b_1)}{b_1^2 e^{b_1}} \quad (20)$$

$$\frac{\sum_{i=1}^4 [\bar{T}(t_i) - \bar{T}(t_{i+4})]^2}{8T_1^2} = \frac{\text{sh}^2(b_1) + \sin^2(b_1)}{2b_1^2 [\text{ch}(b_1) + \cos(b_1)]^2} \quad (21)$$

As opposed to the previously developed procedures [9], here, to determine the thermal diffusivity coefficient  $k$ , one should know in advance the temporal distribution of temperature  $T(y, t_i^*)$ , in the soil layer  $[0, L]$  at an arbitrary dimensionless depth  $y_* = x/L$  and  $T(t_i)$  for *eight time points*; this distribution enables determining the parameter  $k$  by the equation (18)-(21) with higher accuracy.

*The parameters of soil surface temperature*

To determine the soil surface parameters in (2), one and two harmonics were used. By using the measurement results and by using the least square method we determined the parameters of the surface temperature distribution of the soils under study. The preliminary calculation results and their comparison with the experimental data show that the introduction of the second harmonic enables more accurate determination of the temperature

distribution parameters on the soil surface. In future we plan a more detailed study of this procedure application in calculating soil temperature regime and determining thermophysical parameters and characteristics (soil thermal diffusivity coefficient and its dependence on moisture content).

#### CONCLUSIONS

Based on the study of heat transfer model in soil taking into account the boundary conditions dynamics on the surface described by two harmonics, the following was obtained:

point and mean integral solutions;

the theoretical foundations of the procedures for determining soil thermal diffusivity were proposed.

In the future we plan to experimentally test the adequacy of the proposed procedures and compare them with the existing procedures.

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#### APPENDIX A. FORMULAS

$$T(y, t_i) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4}i + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2}i + \alpha_2\right), \quad (i = \overline{1, 8}) \quad (13)$$

$$\begin{aligned} i = 1: \quad T(y, t_1) &= T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 1 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 1 + \alpha_2\right) \\ &= T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} + \alpha_1\right) - \Phi_2(y, b_2) \cdot \sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} i = 2: \quad T(y, t_2) &= T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 2 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 2 + \alpha_2\right) \\ &= T_0 - \Phi_1(y, b_1) \cdot \sin(\alpha_1) - \Phi_2(y, b_2) \cdot \cos(\alpha_2) \end{aligned}$$

$$i = 3: \quad T(y, t_3) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 3 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 3 + \alpha_2\right)$$

$$= T_0 - \Phi_1(y, b_1) \cdot \sin\left(\frac{\pi}{4} + \alpha_1\right) + \Phi_2(y, b_2) \cdot \sin(\alpha_2)$$

$$i = 4: \quad T(y, t_4) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 4 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 4 + \alpha_2\right)$$

$$= T_0 - \Phi_1(y, b_1) \cdot \cos(\alpha_1) + \Phi_2(y, b_2) \cdot \cos(\alpha_2)$$

$$i = 5: \quad T(y, t_5) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 5 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 5 + \alpha_2\right)$$

$$= T_0 - \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} + \alpha_1\right) - \Phi_2(y, b_2) \cdot \sin(\alpha_2)$$

$$i = 6: \quad T(y, t_6) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 6 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 6 + \alpha_2\right)$$

$$= T_0 + \Phi_1(y, b_1) \cdot \sin(\alpha_1) - \Phi_2(y, b_2) \cdot \cos(\alpha_2)$$

$$i = 7: \quad T(y, t_7) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 7 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 7 + \alpha_2\right)$$

$$= T_0 + \Phi_1(y, b_1) \cdot \sin\left(\frac{\pi}{4} + \alpha_1\right) + \Phi_2(y, b_2) \cdot \sin(\alpha_2)$$

$$i = 8: \quad T(y, t_8) = T_0 + \Phi_1(y, b_1) \cdot \cos\left(\frac{\pi}{4} \cdot 8 + \alpha_1\right) + \Phi_2(y, b_2) \cdot \cos\left(\frac{\pi}{2} \cdot 8 + \alpha_2\right)$$

$$= T_0 + \Phi_1(y, b_1) \cdot \cos(\alpha_1) + \Phi_2(y, b_2) \cdot \cos(\alpha_2)$$

Subtracting  $T(y, t_i) - T(y, t_{i+4})$  for the  $i = 1, 2, 3, 4$ , we obtain:

$$1. \quad T_1 - T_5 = 2\Phi_1(y, b_1) \cdot \cos(\pi/4 + \alpha_1) \quad 2. \quad T_2 - T_6 = -2\Phi_1(y, b_1) \cdot \sin(\alpha_1)$$

$$3. \quad T_3 - T_7 = -2\Phi_1(y, b_1) \cdot \sin(\pi/4 + \alpha_1) \quad 4. \quad T_4 - T_8 = -2\Phi_1(y, b_1) \cdot \cos(\alpha_1),$$

and adding the value of their squares, we have:

$$\begin{aligned} \sum_{i=1}^4 [T(y, t_i) - T(y, t_{i+4})]^2 &= 4[\Phi_1(y, b_1)]^2 \cdot \cos^2\left(\frac{\pi}{4} + \alpha_1\right) + 4[\Phi_1(y, b_1)]^2 \cdot \sin^2(\alpha_1) + \\ &\quad + 4[\Phi_1(y, b_1)]^2 \cdot \sin^2\left(\frac{\pi}{4} + \alpha_1\right) + 4[\Phi_1(y, b_1)]^2 \cdot \cos^2(\alpha_1) = \\ &= 4[\Phi_1(y, b_1)]^2 \left\{ \cos^2\left(\frac{\pi}{4} + \alpha_1\right) + \sin^2(\alpha_1) + \sin^2\left(\frac{\pi}{4} + \alpha_1\right) + \cos^2(\alpha_1) \right\} \\ &= 4[\Phi_1(y, b_1)]^2 \{1 + 1\} = 8[\Phi_1(y, b_1)]^2 \end{aligned}$$

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## ТҮЙІН

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### ТОПЫРАҚТАҒЫ ЖЫЛУ ЖЫЛЖУЫНЫҢ ТІКЕЛЕЙ ЖӘНЕ КЕРІ МІНДЕТТЕРІ

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Бұл жұмыста жылу жылжудың кері міндеттерінің теңестірілуінің шешімі негізінде, екі үйлесімділігімен сипатталған беткі қабаттың шекаралық шарттарын есепке ала отырып топырақтағы жылу өткізгіштік коэффициентін анықтау тәсілдері жасалды. Бұл тәсілдермен топырақтағы жылу режимінің математикалық модельдерін қолдану шекараларын кеңейтуді және сәйкестігін көтеретін топырақтағы жылу өткізгіштікті табиғи жағдайда бағалауға болады.

*Түйінді сөздер:* топырақ, моделдеу, жылудың жылжуы, шекаралық шарттар.

## РЕЗЮМЕ

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### ПРЯМЫЕ И ОБРАТНЫЕ ЗАДАЧИ ПЕРЕНОСА ТЕПЛА В ПОЧВЕ

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В работе разработаны методики определения коэффициента температуропроводности почвы, основанные на решении обратных задач уравнения теплопереноса, при учете граничных условий на поверхности, описываемых двумя гармониками. Эти методы позволяют оценивать температуропроводность в почве в естественных условиях, что должно увеличить адекватность и расширить границы использования математических моделей теплового режима почв.

*Ключевые слова:* почва, моделирование, перенос тепла, граничные условия.